

# MEASURES OF CENTER AND VARIATION WORKSHOP

## FULL NOTES

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### PSSM GRADES 6-8

- FIND, USE AND INTERPRET MEASURES OF CENTER AND SPREAD
- DISCUSS AND UNDERSTAND DATA SETS AND CORRESPONDING GRAPHICAL REPRESENTATION

UNDERSTAND MEAN AS 'EVENS OUT' OR 'BALANCES' SET OF DATA

COMPARE UTILITY OF MEAN VS MEDIAN

- \* HOW DO CHANGES IN DATA VALUES AFFECT THE MEAN AND MEDIAN OF A SET OF DATA?

WHAT CHARACTERISTIC DOES A DATA VALUE CHANGE NEED TO HAVE AN AFFECT ON THE MEAN? ON THE MEDIAN?

THE MEAN IS MORE LIKELY TO BE INFLUENCED BY EXTREME VALUES; THE MEDIAN INVOLVES ONLY THE RELATIVE POSITIONS OF THE VALUES

### PSSM GRADES 9-12

- UNIVARIATE: DISPLAY DISTRIBUTION, DESCRIBE SHAPE, CALCULATE STATISTICS
- BIVARIATE: SCATTERPLOT, DESCRIBE SHAPE, DETERMINE REGRESSION AND CORRELATION (WITH TECHNOLOGY)
- DISPLAY/DISCUSS BIVARIATE DATA WITH ONE OR MORE CATEGORICAL
- LINEAR TRANSFORMATIONS AND AFFECT OF SHAPE/CENTER/SPREAD
- IDENTIFY TRENDS AND FIND FUNCTIONS THAT MODEL OR TRANSFORM SO CAN BE MODELED

MULTIPLE MEANINGS OF CENTER WITHOUT SPREAD [DESIGN A DATA SET OF  $x$  ELEMENTS WITH MEAN =  $\_\_\_$  AND MEDIAN =  $\_\_\_$ ]

RECOGNIZE MEAN DOES NOT EQUAL MEDIAN WHEN SAMPLE DATA IS SKEWED

ADDING CONSTANT CHANGES CENTER NOT SPREAD

MULTIPLYING CONSTANT CHANGES CENTER AND SPREAD

# **MEASURES OF CENTER AND VARIATION WORKSHOP FULL NOTES**

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## **NAVIGATING THROUGH DATA ANALYSIS AND PROBABILITY IN GRADES 3-5**

### **HOW MANY STARS CAN YOU DRAW IN ONE MINUTE?**

**CREATE LINE PLOTS, DESCRIBE SHAPE OF DATA, SUMMARIZE DATA,  
COMPARE TWO SETS OF DATA  
ESTIMATING AND PREDICTING  
OUTLIERS AND MEDIAN**

### **DO YOU GET ENOUGH SLEEP?**

**IDENTIFY AND USE MEDIAN, COMPARE MEDIANS OF TWO SETS OF DATA  
LINE PLOTS, 'TYPICAL' VALUE  
GIVEN MEDIAN, WHAT DOES IT TELL YOU ABOUT THE DATA SET?**

### **EXPLORING THE MEAN – HOW MUCH SLEEP DO CHILDREN TYPICALLY GET?**

**IDENTIFY THREE TYPES OF AVERAGE: MEAN, MEDIAN, MODE  
MEAN AS A BALANCE POINT OF A DISTRIBUTION  
CREATE DATA SETS FOR A GIVEN MEAN  
ESTIMATE MEAN OF SMALL SETS OF DATA  
CUBES TO REPRESENT DATA VALUES ON LINE PLOT, SHIFT AROUND**

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## **DATA ANALYSIS AND PROBABILITY ADDENDA SERIES**

### **A LOOK AT THE AVERAGE WAGE**

**MEAN, MEDIAN, AND MODE FROM FREQUENCY TABLE**

**CHANGING DATA VALUES AND REFLECTED CHANGE IN MEASURES OF  
CENTRAL TENDENCY**

**SPREADSHEET EXTENSION**

**USING MEASURES OF CENTRAL TENDENCY FOR ARGUMENTS**

### **EXPLORING STANDARD DEVIATION**

**USING CALCULATOR TO DETERMINE MEASURES OF CENTRAL TENDENCY  
AND STANDARD DEVIATION**

**COMPARING STATISTICS FROM DIFFERENT DATA SETS**

**EXAMINING SHAPE OF DATA (MEAN CONSTANT)**

**GIVING CONTEXT TO DATA**

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### NAVIGATING THROUGH DATA ANALYSIS IN GRADES 6-8

#### TV WATCHING

DO MIDDLE SCHOOL STUDENTS WATCH TOO MUCH TV?

MEAN/MEDIAN/MODE, PERSPECTIVE (STUDENT AND PARENT), SUBJECTIVE JUSTIFICATION

#### MAKING THE DATA

CREATING DATA SETS WITH GIVEN MEAN/MEDIAN/RANGE VALUES, STRATEGIES

HOW MANY DATA SETS POSSIBLE?

'BALANCE' AROUND THE MEAN [INTRODUCES A VERSION OF WEIGHTED MEAN]

#### DROP OFF

INTERPRET HISTOGRAM, IDENTIFY MEASURE OF CENTRAL TENDENCY, MAKE INFERENCES

#### STUDENTS AND BASKETBALL PLAYERS

STEM-AND-LEAF PLOTS AND 'TYPICAL' HEIGHT OF STUDENTS

"HALF" "MOST" EXPLANATION

DIFFERENCES IN MEANS

#### STOPPING DISTANCES (OF TWO CAR MODELS AT TWO SPEEDS)

APPROPRIATE REPRESENTATION, EXPLANATION

GROUP WORK

NEGOTIATE POSITION AND JUSTIFY STANCE

MEAN/MEDIAN/MODE/RANGE OR LINE PLOTS

CONSISTENCY, (SAMPLE SIZE)

CONSIDERATION OF RANGE

#### BATTERIES

IDENTIFY AND COMPARE CHARACTERISTICS OF DATA DISTRIBUTIONS

SELECTING GRAPHIC REPRESENTATION

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## **NAVIGATING THROUGH DATA ANALYSIS IN GRADES 9-12**

### **SAMPLING RECTANGLES**

**AREA OF 'TYPICAL' RECTANGLE VS 'RANDOM' RECTANGLE**

**SAMPLING DISTRIBUTIONS, SIMPLE RANDOM SAMPLE, VARIABILITY, BIAS**

**MEASURES OF CENTER AND SPREAD**

**TYPES OF SAMPLES AND RESPONSES**

**SAMPLE VS POPULATION**

### **SAMPLE SIZE**

**SIMILAR TO PREVIOUS WITH SAMPLE SIZE OF 10**

**(NO MORE THAN 10% OF POPULATION)**

**INCREASED SAMPLE SIZE MEANS DECREASED VARIABILITY**

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**MEAN AND MEDIAN: ARE THEY REALLY SO EASY?, *MTMS*, MARCH 2000**

**PROCEDURAL VS CONCEPTUAL UNDERSTANDING**

**STUDENTS TEND TO SELECT MEAN OVER MEDIAN TO REPRESENT A DATA SET  
REGARDLESS OF DISTRIBUTION**

**NAEP ITEMS: MEAN/MEDIAN/MODE, MEDIAN FROM SCATTERPLOT, BEST  
MEASURE**

**IS AVERAGE BETTER UNDERSTOOD THAN MEAN?**

**IDEA THAT MEAN IS MORE PRECISE/ACCURATE BECAUSE IT INCLUDES ALL OF  
THE NUMBERS AND MEDIAN IS JUST ONE NUMBER**

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DEVELOPING A MEANINGFUL UNDERSTANDING OF THE MEAN, *MTMS*,  
SEPTEMBER 2003

WHEN INTRODUCED, MEAN IS OFTEN PRESENTED AS AN APPLICATION OF  
DIVISION RATHER THAN A STATISTICAL CONCEPT

POSSIBILITY EXISTS FOR STUDENTS TO COMPUTE THE MEAN WITHOUT  
DEVELOPING AN UNDERSTANDING OF WHAT THE MEAN  
REPRESENTS, HOW IT IS RELATED TO THE NUMBERS IN THE  
DATA, OR HOW IT IS RELATED TO OTHER MEASURES OF THE  
CENTER OF SPREAD OF A DATA SET – AS A PROCEDURE

SEVEN PROPERTIES OF THE MEAN

1. THE MEAN IS LOCATED BETWEEN EXTREME VALUES
2. THE SUM OF THE DEVIATIONS FROM THE MEAN IS ZERO
3. THE MEAN IS INFLUENCED BY VALUES OTHER THAN THE MEAN
4. THE MEAN DOES NOT NECESSARILY EQUAL ONE OF THE VALUES THAT WAS SUMMED
5. THE MEAN CAN BE A FRACTION THAT HAS NO COUNTERPART IN PHYSICAL REALITY
6. WHEN ONE CALCULATES THE MEAN, A VALUE OF ZERO, IF IT APPEARS, MUST BE TAKEN INTO ACCOUNT
7. THE MEAN VALUE IS REPRESENTATIVE OF THE VALUES THAT WERE AVERAGED

‘FUND-RAISING CONTEST’ PROBLEM

BEST PERFORMANCE AMONGST GROUPS OF DIFFERENT SIZE

DISCUSSION OF MEAN AND RANGE

SUM OF DEVIATIONS FROM MEAN EQUAL 0

‘WHAT HAPPENS IF . . .?’ PROBLEM

PROBLEM WITHOUT CONTEXT, ASKS STUDENTS TO DEVELOP

MEAN VALUE CAN HAVE NO MEANING IN CONTEXT

HOW ADDING VALUES TO A DATA SET AFFECTS THE MEAN

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**MATH PHOBIA MEDIA CLIPS**

**TOTAL TIME OF TRAIN TRAVEL WITH A 10% REDUCTION IN SPEED AFTER EVERY  
STOP**

**CREATE DATA SETS WITH MEAN = MEDIAN, MEAN > MEDIAN, AND MEAN <  
MEDIAN**

**MEAN AND MEDIAN INCOMES WITH GROUPS OF PEOPLE INVOLVING BILL GATES  
MEDIAN AS 'MIDPOINT'**

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**UNDERSTANDING THE MEAN AND THE MEDIAN THROUGH AN INTERACTIVE  
ACTIVITY**

**HOW CHANGE(S) IN DATA VALUE(S) AFFECT MEAN/MEDIAN  
WHY?**

**APPLET (STANDARDS CD – SECTION 6.6)**

**WHAT SORT OF DATA CHANGES AFFECT THE MEAN? THE MEDIAN?**

**DESIGN DATA SETS TO GIVEN MEAN AND MEDIAN**

**HOW MANY POSSIBLE?**

**CAN YOU FIND WAYS TO MOVE DATA POINTS TO KEEP THE MEAN/MEDIAN THE  
SAME BUT CHANGE THE MEDIAN/MEAN?**

**JUST AN AVERAGE WORKSHEET**

**WORD LENGTH – MEAN OR MEDIAN – JUSTIFY**

**‘CALCULATE THE CENTER’**

**GIVEN TWO STEM-AND-LEAF PLOTS, WHICH MEASURE OF SUMMARY TO USE**

**INTERPRET MEAN AND MEDIAN OF SKEWED DATA**

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## **TEACHING STATISTICS – WHAT’S AVERAGE?**

**MODE – SOME STUDENTS CONFUSE ‘MOST FREQUENT’ WITH ‘MOST HAVE’**

**ONLY MEASURE OF CENTER THAN CAN BE CATEGORICAL**

**NOT PARTICULARLY USEFUL WITH QUANTITATIVE DATA**

**MEDIAN – PLACING DATA ELEMENTS ON GRID PAPER AND FOLDING IN HALF TO  
REPRESENT AS THE MIDDLE OF AN ORGANIZED DATA SET**

**MOVING TO ABSTRACT THINKING PROCESS**

**STABILITY OF THE STATISTIC**

**MEAN – FAIR SHARE/EVENING OUT STRATEGY OR USING THE BALANCE MODEL**

**BEST USE ACTIVITY – WHEN AND WHY**

## **EXPLORING MEAN, MEDIAN, AND MODE WITH A SPREADSHEET**

**SPREADSHEET ACTIVITIES DESIGNED TO ATTACK TYPICAL STUDENT  
MISCONCEPTIONS**

**‘TYPICAL’ OR AVERAGE STUDENT**

**SPREADSHEET FORMULAS AND FEATURES**

**EXPLORATION, THEN ALGORITHMS**

**TO DETERMINE MEAN HEIGHT, HAVE STUDENTS CUT STRING THE SAME LENGTH  
AS THEIR HEIGHT, TIE THE LENGTHS TOGETHER, THEN FOLD THE  
STRING INTO AS MANY EQUAL LENGTH PARTS AS THE NUMBER  
OF STUDENTS**

# **MEASURES OF CENTER AND VARIATION WORKSHOP FULL NOTES**

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**WHAT DO CHILDREN UNDERSTAND ABOUT AVERAGE?**

**UNDERSTAND PROCEDURE BUT NOT CONCEPT**

**CREATING DATA SET WITH GIVEN AVERAGE**

**STUDENTS TEND TO SELECT A SINGLE VALUE (MODE VIEW)**

**SOME USED AVERAGE AS MIDDLE AND THEN 'BALANCED' SYMMETRICALLY  
(MEDIAN VIEW)**

**FRUSTRATED IF TRIED TO USE ALGORITHM FOR MEAN AND WORK  
BACKWARDS**

**STUDENTS NEED TO UNDERSTAND MEDIAN BEFORE MEAN**

**"FAIR SHARE" MODEL HAD PROBLEM OF LOSING THE ORIGINAL ELEMENTS OF  
DATA**

**"BALANCE" MODEL LOOKS AT DIFFERENCES FROM MEAN, BUT STUDENT MUST  
UNDERSTAND THE CONCEPT OF BALANCE**

**"UNPACKING" MODEL USES LINE PLOTS AND POST-IT NOTES, INCORPORATES  
IDEA OF BALANCE**

**IMPORTANCE OF CONTEXT AND MULTIPLE EXAMPLES**

**ACTION RESEARCH IDEAS OFFERED**

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### CHILDREN'S CONCEPTS OF AVERAGE AND REPRESENTATIVENESS

EDUCATORS ASSUME AVERAGE IS SIMPLY ANOTHER APPLICATION OF DIVISION  
AND IF CHILDREN UNDERSTAND FAIR OR EQUAL SHARE, THEY  
WILL UNDERSTAND THE NOTION OF AVERAGE

EXAMINE HOW CHILD DESCRIBES AND CONSTRUCTS SETS OF DATA WHILE  
EXAMINING HOW THEY UNDERSTAND AVERAGE

STUDENTS DEVELOP THEIR OWN IMPRESSION OF 'AVERAGE' AND TYPICAL  
TWO RESEARCH QUESTIONS (N = 21)

WHEN ASKED TO DESCRIBE A REAL DATA SET, HOW DO CHILDREN  
CONSTRUCT AND INTERPRET REPRESENTATIVENESS?

HOW DO CHILDREN UNDERSTAND THIS MATHEMATICAL OBJECT AND HOW DO  
THEY CONNECT IT WITH THEIR INFORMAL MATHEMATICAL  
UNDERSTANDING?

STUDENT INTERVIEWS AND THINK ALOUD TO EXPLAIN ANSWERS

IDENTIFY FIVE PREDOMINANT APPLICATIONS TO SOLVE PROBLEMS

1. AVERAGE AS MODE
2. AVERAGE AS ALGORITHM
3. AVERAGE AS REASONABLE
4. AVERAGE AS MIDPOINT
5. AVERAGE AS MATHEMATICAL POINT OF BALANCE

LAST THREE APPLICATIONS INDICATE STUDENTS WORKING TOWARD A  
DEFINITION OF AVERAGE

PREMATURE LEARNING OF THE ALGORITHM HURTS CONCEPTUAL  
UNDERSTANDING OF AVERAGE

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WHERE SHOULD IT BE?

# MEASURES OF CENTER AND VARIATION WORKSHOP

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### NOTES:

- CD OF MATERIALS OR REFERENCE [WATCH COPYRIGHT]
- ROLE OF CALCULATOR AND TECHNOLOGY?
- TRACK MMM/SPREAD THROUGH MS & HS TEXTBOOKS: TRADITIONAL AND STANDARDS-BASED
- SCAFFOLDING
- DOES THE PRESENTATION INCLUDE SAMPLING METHODS? IF NOT, REVISE DATA RECORD SHEET IN HS *NAVIGATIONS*.
- FIND MEAN, MEDIAN, AND MODE FROM FREQUENCY TABLE
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